Welcome to Math 41

Instructor: Nick Payne (he/him)

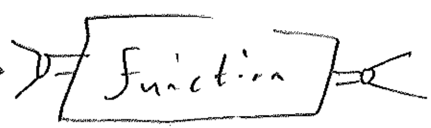
Today: - Introductions
- Syllabus and Canvas
- If time, start Section 1.1

Section 1.1 Basics of Functions

Definition A *function* is a map between two collections of objects (usually numbers).

Another way to say it: A function is a rule that assigns one object in one collection to exactly one object in another collection.

Example 1

input x →  → output y

"the function machine"

inputs	outputs
1 →	5
2 →	-2
3 →	100

$1 \to 3$

$5 \to 3$

$6 \to 7$

An input can only have exactly one output! So, 5 in this example cannot also be mapped to 7.

maps between numbers

TV → 100

Radio → 20

icecream → 5

A price function.

The function $y = 2 \cdot x$ is a rule that assigns every input to two times that input.

We say "y is a function of x"

If $x = 4$, then $y = 2 \cdot 4 = 8$

Def/ To make things easier to understand, we also use functional notation.

That is, $f(x) =$ ______, where x is the independent variable, and the right hand side is the rule.

Ex 2/ $f(x) = x^2$ means when $x = 3$, $f(3) = 9$
$x = -2$, $f(-2) = 4$
$x = a$, $f(a) = a^2$
$x = x+2$, $f(x+2) = (x+2)^2$
$\vdots$

Note: - $f(x)$ means "f evaluated at x".

- f, x are both placeholders (could use $h, g, y, \ldots$)

Ex 3/ $f(x) = 2x$, $h(t) = t^2$.
$g(y) = \frac{1}{2y}$, $y = 5^x$ are all functions
$y = \sqrt{t}/2t^2$

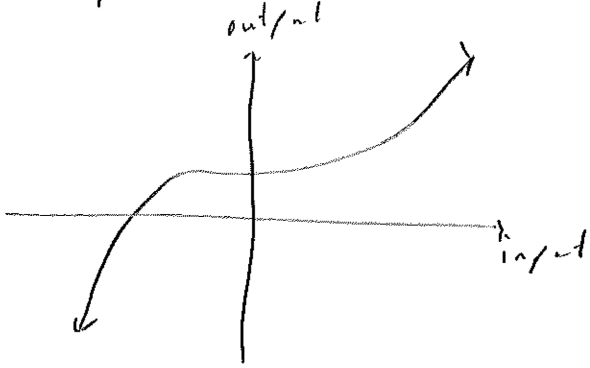
Ex 4/ Let $f(x) = 3\sqrt{x+2}$. Find $f(7)$ and $f(h+1)$

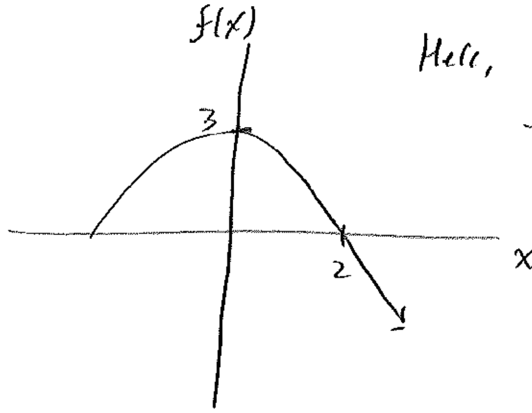
$f(7) = 3 \cdot \sqrt{7+2} = 3 \cdot \sqrt{9} = 3 \cdot 3 = 9$

$f(h+1) = 3\sqrt{(h+1)+2} = 3 \cdot \sqrt{h+3}$.

Other ways of representing functions

Graphs





Here, $f(2)=0$
$f(0)=3$

Tables

t	$V(t)$
0	3.4
20	3.6
40	4.1
60	4.2

Voltage of a battery $V(t)$ in volts after charging for t minutes.

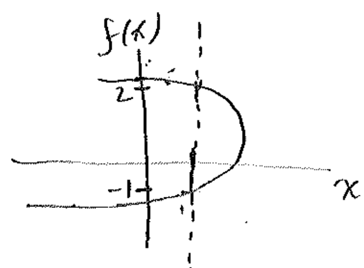
What does $V(40)=4.1$ mean?

After charging for 40 minutes, the battery is at 4.1 volts.

Ex 5

$\boxed{\text{Not functions}}$

$x+y^2-2y=0$



why? $x=0$ gives $y=0$ and $y=2$

☆ $x=1$ gives $f(1)=-1$ and 2

☆ This is called the vertical line test.

If there exists any x such that a vertical line through x hits two (or more) $f(x)$-values then f is not a function.

Important Ideas

Interval notation (P. 2 in textbook) For $a < b$

(a,b): the set of all real numbers between a and b (not including either a nor b)

$(a,b]$: — " —, including b, but not a (similarly for $[a,b)$)

$(-\infty,\infty)$: all real numbers

(a,∞): — " — greater than a

$(-\infty,b)$: — " — less than b

Def/ The <u>domain</u> is the set of all allowable inputs for a function.

Practically, it is the set of inputs that make sense.

Def/ The <u>range</u> is the set of possible outputs for a function.

Ex 6/

f

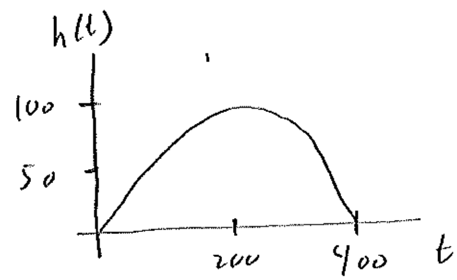
$2 \to 3$

$4 \to 2$

$7 \to 2$

domain: $\{2,4,7\}$

range: $\{2,3\}$



domain: $[0,100]$

range: $[0,400]$

$g(x) = \frac{1}{x+1}$

What are the domain and range?

Strategies

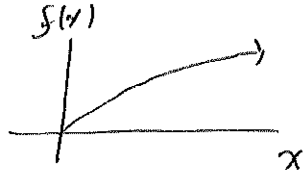
Domain: all real numbers except those that don't make sense, like: $\frac{1}{0}$, $\sqrt{-1}$, 0^0, ...

Ex] the domain of $f(x) = \frac{1}{x}$ is $(-\infty, 0) \cup (0, \infty)$

—— " —— $g(x) = \sqrt{x}$ is $[0, \infty)$.

Range: Use what you know about the function to make a sketch:

Ex] $f(x) = \sqrt{x}$

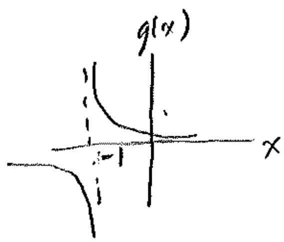


Range is $[0, \infty)$

Back to Ex 6]

$g(x) = \frac{1}{x+1}$. Domain: denominator cannot be 0, so $x \neq -1$.

So, $(-\infty, -1) \cup (-1, \infty)$



Range: $g(x)$ can never be 0 (see sketch)

So, $(-\infty, 0) \cup (0, \infty)$

Def] A <u>piecewise function</u> is a function defined on multiple intervals.

Ex] $f(x) = \begin{cases} -2x, & x < 0 \\ x, & x \geq 0 \end{cases}$