2.1 Function Operations

Constant multiple: $g(x) = r f(x)$, r a constant

Domain of g = domain of f

Ex] $f(x) = x^3 - 2$, $r = 2$

$g(x) = 3f(x) = 2x^3 - 4$

Addition: $h(x) = g(x) + f(x)$

Domain of h = dom. of g intersect dom. of f

Ex] $f(x) = x^2$, $g(x) = 2x + 1$

$h(x) = f(x) + g(x) = x^2 + 2x + 1$

Multiplication: $h(x) = g(x) \cdot f(x)$

Domain of h = domain of g intersect domain of f.

Ex] $f(x) = x^2$, $g(x) = 2x + 1$

$h(x) = f(x)\, g(x) = 2x^3 + x^2$

Division: $h(x) = \dfrac{f(x)}{g(x)}$

Domain of h = domain of g intersect domain of g, excluding any x such that $g(x) = 0$.

Ex] $f(x) = x^2$, $g(x) = 2x + 1$

$h(x) = \dfrac{f(x)}{g(x)} = \dfrac{x^2}{2x+1}$

Domain of f: $(-\infty, \infty)$

Domain of g: $(-\infty, \infty)$

Domain of h: $(-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, \infty)$

Notation: $f(x)+g(x) \Longleftrightarrow (f+g)(x)$

$f(x)g(x) \Longleftrightarrow (fg)(x)$

Composition: Let f, g be functions.

Def] $h = f \circ g$ is the composition of f with g, with $h(x) = (f \circ g)(x) = f(g(x))$

To find $h(x)$, first calculate $g(x)$. Then, plug in $g(x)$ to f.

Remember: $f(g(x))$ just means: anytime you see an x in $f(x)$, plug in $g(x)$.

Domain of h is all x in the domain of $g(x)$ for which $g(x)$ is in the domain of f.

Ex] $f(x) = x^2 + x$, $g(x) = 4x$

$(f \circ g)(x) = f(g(x)) = (g(x))^2 + g(x) = (4x)^2 + 4x = 16x^2 + 4x$

Domain of g is $(-\infty, \infty)$. all x in $(-\infty, \infty)$ give a $g(x)$ which is in domain of f.

So, domain of $(f \circ g)(x)$ is $(-\infty, \infty)$.

Ex] $f(x)=\frac{1}{x}$, $g(x)=x-1$.

$$(f\circ g)(x)=f(g(x))=\frac{1}{g(x)}=\frac{1}{x-1}.$$

Domain of g is $(-\infty,\infty)$. But, domain of f is all x except 0. So, $g(x)$ cannot be 0. Where is $g(x)=0$? at $x=1$. So,

Domain of $(f\circ g)(x)$ is $(-\infty,1)\cup(1,\infty)$.

Decomposition: going the opposite direction.

Ex] $h(x)=\sqrt{x^2+1}$. What are possible $f(x)$ and $g(x)$ such that $h(x)=(f\circ g)(x)$?

$f(x)=\sqrt{x}$, $g(x)=x^2+1$.

Even/Odd functions] (Not in textbook)

A function $f(x)$ is even if $f(x)=f(-x)$
—— || —— odd if $f(-x)=-f(x)$

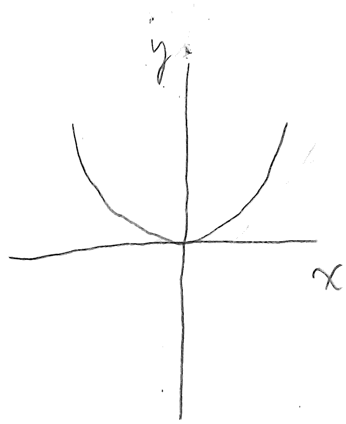
Ex] $f(x)=x^2$ is even, because $f(-x)=(-x)^2=x^2=f(x)$
$g(x)=x^3$ is odd because $g(-x)=(-x)^3=-x^3=-g(x)$.

Graphically:

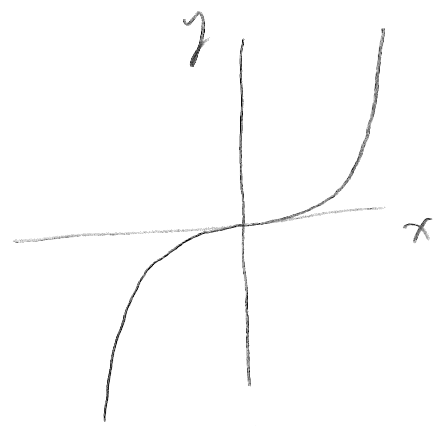
Even functions are symmetric about the y-axis.

Odd functions are symmetric about the origin
(i.e. it looks the same from any two opposite directions.)

Ex



Even



odd

Note: Most functions are neither odd nor even.