2.2 Inverse Functions

Inverse "=" undoing the function.

Ex] function tells you your height given your age, the inverse tells you your age given your height

Def] g is the <u>inverse function</u> of f if $g(f(x))=x$ for all x in the domain of f and $f(g(y))=y$ for all y in the range of f.

OR: g is the inverse of f if

$g(y)=x$ if and only if $f(x)=y$

Notation: inverse of f = f^{-1}

Ex] Let $f(x)=x+1$, $g(x)=x-1$

$$f(g(x)) = g(x)+1 = (x-1)+1 = x$$
$$g(f(x)) = f(x)-1 = (x+1)-1 = x$$

$\rightarrow$ $g=f^{-1}$ (and $f=g^{-1}$)

Ex]

x	$f(x)$
0	3
2	-1
5	7
9	2

Find $f^{-1}(2)$

$f^{-1}(2) = 9$

Ex] Domain $f = [-1, 1]$, Range $f = [2, 3]$

Domain $f^{-1} = [2, 3]$, Range $f^{-1} = [-1, 1]$

Finding inverses. Let $y = f(x)$ be a function.

~~To find $g = f^{-1}$~~, we solve for x in terms of y.

Ex] $y = 3x - 1 \longrightarrow x = \frac{y+1}{3} \longrightarrow$ inverse is $\frac{y+1}{3}$

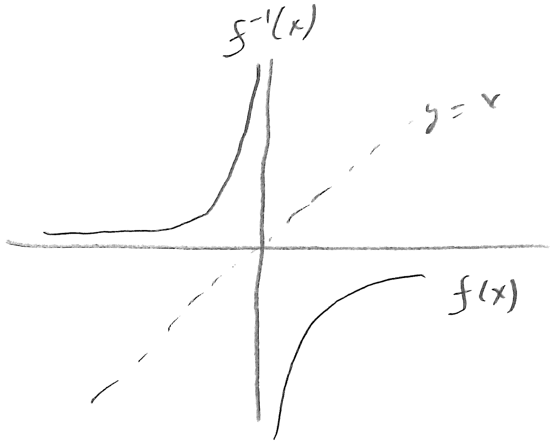
or, $f^{-1}(y) = \frac{y+1}{3}$

Variables don't matter, so $f^{-1}(x) = \frac{x+1}{3}$ is another way to write f^{-1}.

Alternatively, if $y(x) = 3x - 1$, then $x(y) = \frac{y+1}{3}$.

Graphically, the inverse is the reflection about $y=x$.

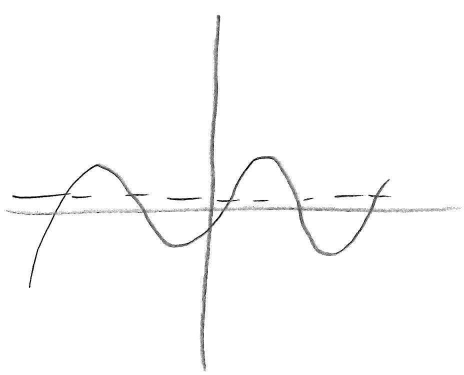
Ex]



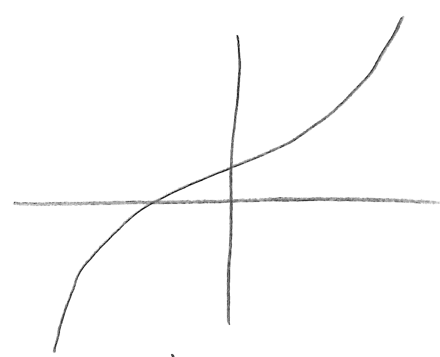
⚠ Not all functions have inverses

Horizontal line test: f^{-1} exists if each horizontal line meets the graph of $f(x)$ no more than once.

Ex]



No inverse exists

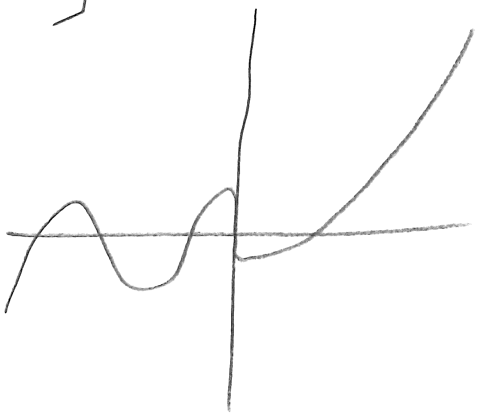


inverse exists

Def] An invertible function is also called a <u>one-to-one</u> function

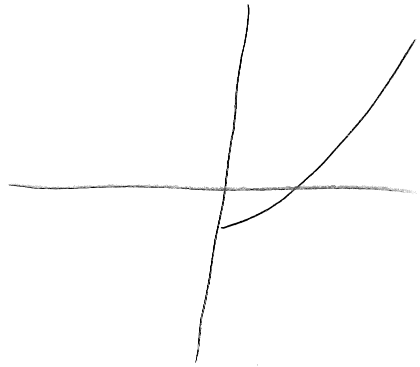
Sometimes, a function only has an inverse if we shrink the domain.

Ex]



Domain $(-\infty, \infty)$

No inverse exists



Domain: $(0, \infty)$

Inverse exists

Ex] $f(x) = x^2$. with domain $(-\infty, \infty)$, f is not one-to-one.

$h(x) = x^2$, domain $[0, \infty)$, h is one-to-one.

Solve for h^{-1}: $h(x) = y = x^2 \longrightarrow \sqrt{y} = x$

$\longrightarrow h^{-1}(x) = \sqrt{x}$.