4.2 Laws of Logarithms

Recall: $e^{A+B} = e^A e^B$, $e^{pA} = (e^A)^p$, $e^{A-B} = \frac{e^A}{e^B}$ f.l any A, B.

Let $A = \ln x$, $B = \ln y$

Then,

- $e^{\ln x + \ln y} = e^{\ln x} e^{\ln y} = xy$

  i.e. $\ln(xy) = \ln(x) + \ln(y)$

- $e^{p \ln x} = (e^{\ln x})^p = x^p$

  i.e. $\ln(x^p) = p \ln x$

- $e^{\ln x - \ln y} = \frac{e^{\ln x}}{e^{\ln y}} = \frac{x}{y}$

  i.e. $\ln\left(\frac{x}{y}\right) = \ln x - \ln y$.

Generalizing,

$$\boxed{\begin{aligned} \log_b(xy) &= \log_b x + \log_b y \\ \log_b\left(\frac{x}{y}\right) &= \log_b x - \log_b y \\ \log_b(x^p) &= p \log x \end{aligned}}$$

Ex]
$$\log_6 4 + \log_6 9 = \log_6 (4 \cdot 9)$$
$$= \log_6 (36)$$
$$= 2$$

$$\log_2 48 - \log_2 3 = \log_2 \left(\frac{48}{3}\right)$$
$$= \log_2 (16)$$
$$= 4$$

$$\log x - \log(x^2+1) = \log\left(\frac{x}{x^2+1}\right)$$

$$y = \ln 2^x = 2 \ln x$$

Glumpwalk: $\cdot \ln \left(\frac{e^3}{(3-2x)^4}\right)$

- $\log_2 (x-3) + \log_2 (5x+1) - \log_2 (x)$

- $2 \ln x - \ln (1+x) + 3 \ln (3-17x)$