4.3 Solving Equations

Examples (1) $5^x = 17$

$\ln(5^x) = \ln(17)$

$x \ln(5) = \ln(17)$

$x = \frac{\ln(17)}{\ln(5)}$

Note: It doesn't matter which base we choose!

By change of base,

$x = \frac{\ln(17)}{\ln(5)} = \frac{\log_5(17)/\log_5(e)}{\log_5(5)/\log_5(e)}$

$= \frac{\log_5(17)}{\log_5(5)}$

For any b.

(2) $6e^{-x} = e^{2x}$

$6 = \frac{e^{2x}}{e^{-x}} = e^{2x} e^{x} = e^{3x}$

$\ln(6) = \ln(e^{3x})$

$\ln(6) = 3x \ln e$

$x = \frac{\ln(6)}{3}$

(3) $2^{3x} = 5 \cdot 4^x$

$(2^3)^x = 5 \cdot 4^x$

$8^x = 5 \cdot 4^x$

$\frac{8^x}{4^x} = 5$

$2^x = 5$

$\ln(2^x) = \ln 5$

$x \ln(2) = \ln 5$

$x = \ln 5 / \ln 2$

or

$\ln 2^{3x} = \ln(5 \cdot 4^x)$

$3x \ln 2 = \ln 5 + \ln 4^x$

$3x \ln 2 = \ln 5 + x \ln 4$

$3x \ln 2 - x \ln 4 = \ln 5$

$x(3 \ln 2 - \ln 4) = \ln 5$

$x = \frac{\ln 5}{3 \ln 2 - \ln 4}$

(4) $\log(2x+4) = \log(x+20)$

$10^{\log(2x+4)} = 10^{\log(x+20)}$

$2x+4 = x+20$

$x = 16$

We always choose the base of the exponential to match the base of the logarithm.

Here, we choose 10 because for $\log_{10}$, $b = 10$.

(5) $\ln x + \ln(x+2) = \ln 3$

$\ln(x(x+2)) = \ln 3$

$e^{\ln(x(x+2))} = e^{\ln 3}$

$x^2 + 2x = 3$

$x^2 + 2x - 3 = 0$

$x = -3$, or $x = 1$

Notice that a possible $x(-3)$ is not in the domain of $\ln x$. Therefore, only $x = 1$ is a solution!

(6) $2 = \ln(1+3^x)$

$e^2 = e^{\ln(1+3^x)}$

$e^2 = 1+3^x$

$e^2 - 1 = 3^x$

$\ln(e^2-1) = \ln 3^x$

$\ln(e^2-1) = x \ln 3$

$x = \dfrac{\ln(e^2-1)}{\ln 3}$

Sometimes, we need both exponentials and logarithms.

(7) An investment B grows according to the formula $B(t) = 800(1.07^t)$, where t is in years and B is in dollars.

How long does it take B(t) to double?

$B_0 = 800$, → when doubled, we have \$1600. So,

$1600 = 800(1.07^t)$

$2 = 1.07^t$

$\ln(2) = \ln(1.07^t)$

$\ln(2) = t \ln(1.07)$

$t = \dfrac{\ln 2}{\ln(1.07)} \approx 10.24$ years

How long does it take t to double again?
(Answer: the same).