5.1 Quadratic Functions

Def] Polynomial of degree n is a function:

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

with $a_0, \dots a_n$ constants, $a_n \neq 0$.

(1) degree: n
(2) leading term: $a_n x^n$
(3) leading coefficient: a_n
(4) constant: a_0

Def] A zero of a polynomial $P(x)$ is a number c such that $P(c) = 0$.

Def] A quadratic function is of the form

$$f(x) = ax^2 + bx + c, \quad a \neq 0.$$

To solve $f(x) = 0$, use quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Def] The discriminant is $d = b^2 - 4ac$

Ex/ Find the discriminant of $f(x) = x^2 - \sqrt{8}x + 2$.

$$d = b^2 - 4ac = (-\sqrt{8})^2 - 4(1)(2) = 0$$

Note: $d > 0$ means $p(x)$ has two real zeros

$d = 0$ ——— " ——— one real zero

$d < 0$ ——— " ——— two complex zeros.

Recall: complex numbers

$i = \sqrt{-1}$, $i^2 = -1$,

A complex number is of the form $z = a + bi$, where a, b are real numbers

The complex conjugate is $\bar{z} = a - bi$.

Note: If $p(x)$ has two complex roots (i.e. $d < 0$), then its zeros occur in complex conjugate pairs.

Ex/ $f(x) = 2x^2 - 4x + 4$. Find roots of $f(x)$.

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(4)}}{2(2)} = \frac{4 \pm \sqrt{16 - 32}}{4}$$

$$= \frac{4 \pm \sqrt{-16}}{4} = \frac{4 \pm 4i}{4} = 1 \pm i.$$

Factor Theorem

If $x=p$ and $x=q$ are distinct zeros of a quadratic polynomial $p(x)$ with leading coefficient a, then:

$$p(x) = a(x-p)(x-q)$$

If p is the only zero, then:

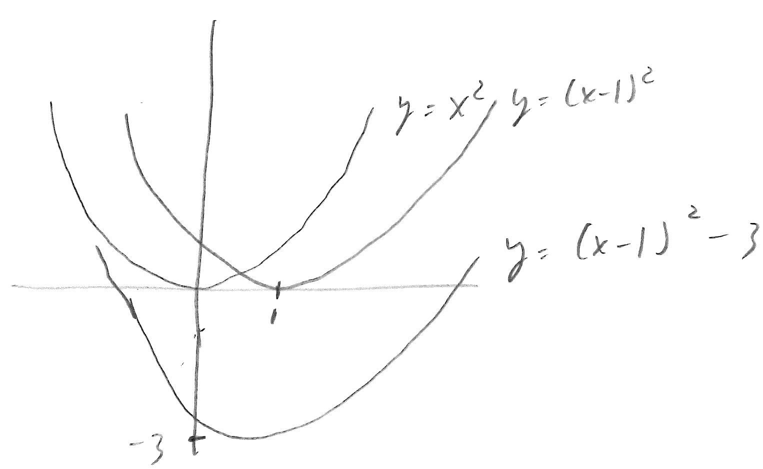
$$p(x) = a(x-p)^2$$ p is a repeated root.

Ex] factor $f(x) = 2x^2 - 4 + 4$

by before, $f(x) = 2\big(x - (1-i)\big)\big(x - (1+i)\big)$

Graphing] Always compare to $y = x^2$.

Ex] sketch $y = \underbrace{(x-1)}_{\text{shifts } x \text{ by } 1}{}^2 - 3$ ← shifts y by 3



Def] $P(x)$ is in vertex form if

$$P(x) = a(x-h)^2 + k$$

The vertex of $P(x)$ is (h, k)

For a quadratic in the form

$$f(x) = ax^2 + bx + c,$$

(1) The vertex occurs at $x = -\frac{b}{2a}$

(2) The vertex is a minimum of $f(x)$
if $a > 0$

—— " —— maximum —— " —— $a < 0$.

Ex) A projectile is launched upwards. After t seconds, its height h is modeled as

$$h(t) = -16t^2 + 64t + 6.$$

1. What is the initial height of the projectile?

(think $t = 0$) $h(0) = 6$ ft.

2. At what time does the projectile hit the ground?

$h(t) = 0 = -16t^2 + 64t + 6$

Use quadratic formula.

$$t = \frac{-64 \pm \sqrt{(64)^2 - 4(-16)(6)}}{2(-16)}$$

$$= \frac{-64 \pm \sqrt{4480}}{-32}$$

$$= \frac{-64 \pm 67}{-32}$$

$$\approx \underbrace{-.09}_{\text{outside of domain.}},\ 4.09$$

$t = 4.1$ seconds

3. What is the maximum height that will be reached? When does this occur?

Use vertex form! $t = -\frac{b}{2a} = \frac{-64}{2(-16)} = 2$.

$$h(2) = -16(2)^2 + 64(2) + 6$$
$$= 70\,ft$$

4. What is a reasonable domain and range?

Domain: $[0, 4.1]$

Range: $[0, 70]$

5. Sketch $h(t)$