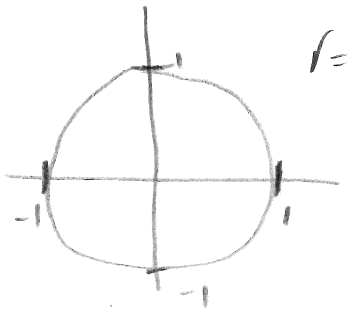
6c.2 Unit Circle

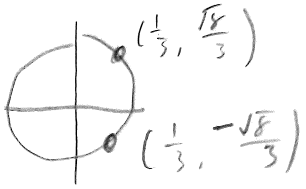
Def Unit circle



$x^2 + y^2 = 1$

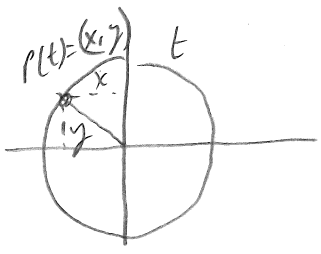
Ex If $(\frac{1}{3}, y)$ lies on the unit circle, what is y?

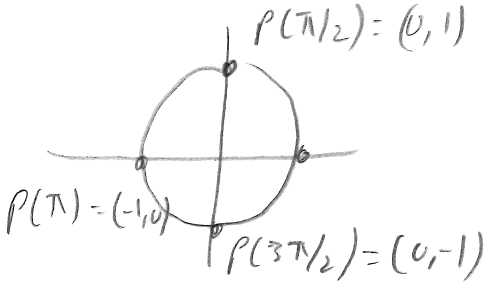
$$1 = x^2 + y^2 = \left(\frac{1}{3}\right)^2 + y^2 \longrightarrow y^2 = \frac{8}{9} \longrightarrow y = \pm\frac{\sqrt{8}}{3}$$

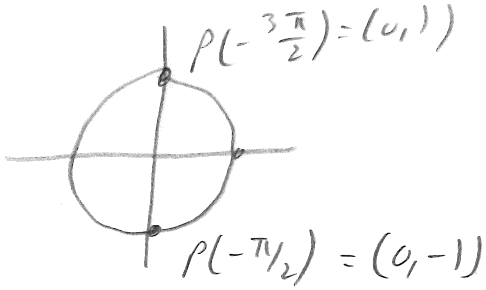


Def Trigonometric point $P(t)$ is the point after travelling t radians around the circle, starting at $(1,0)$

Ex

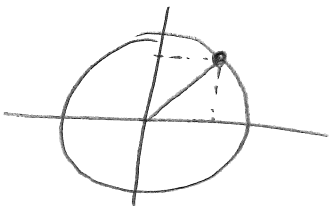






Ex) How to calculate $P(\frac{\pi}{4})$?

$\pi/4$ radians $= 45°$



$y = x$ is a line making an angle of $45°$.

Let $P(\frac{\pi}{4}) = (x, y)$. We know that

$x^2 + y^2 = 1$ and $y = x$.

So, $1 = x^2 + y^2 = x^2 + x^2 = 2x^2$

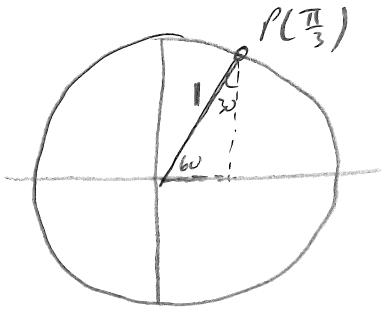
$\longrightarrow x^2 = \frac{1}{2} \rightarrow x = \pm \frac{1}{\sqrt{2}}$.

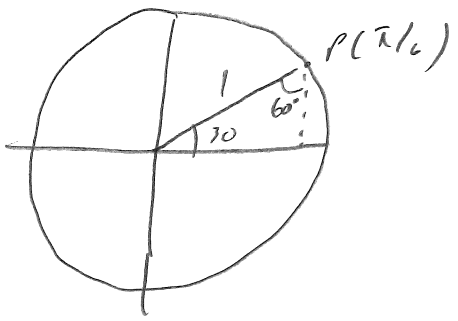
Here, x is positive. So,

$$P(\tfrac{\pi}{4}) = (\tfrac{1}{\sqrt{2}}, \tfrac{1}{\sqrt{2}})$$

Ex) What about $P(\frac{\pi}{3})$ or $P(\frac{\pi}{6})$?

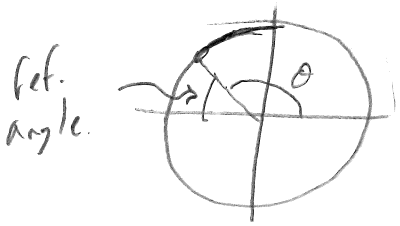
$\frac{\pi}{3}$ rad $= 60°$, $\frac{\pi}{6}$ rad $= 30°$





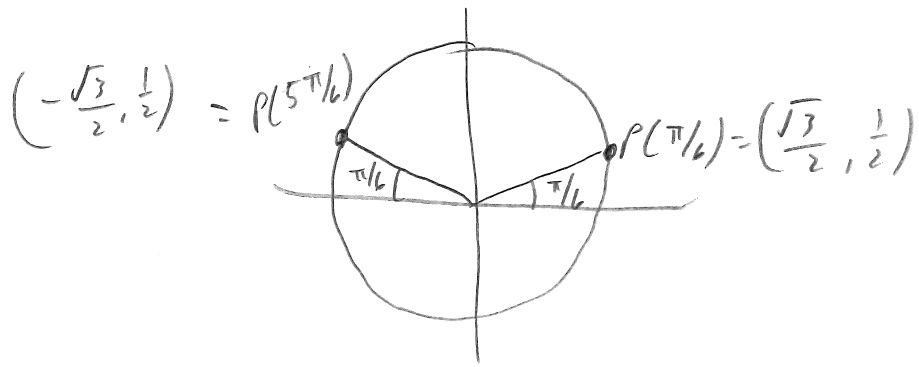
Def Reference angle: smallest angle to x-axis

Ex

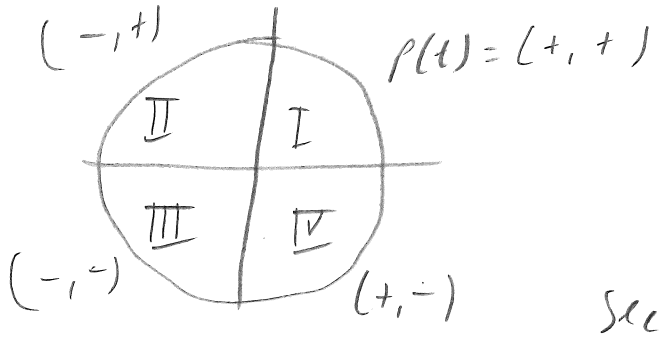


Q: Why are reference angles useful?

A: If we know $P(0)$, $P(\pi/6)$, $P(\pi/4)$, $P(\pi/3)$, we can fill out points along the entire unit circle



In general:



See Table 6C.3