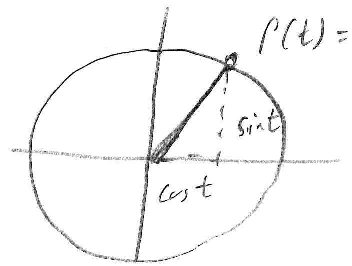
6C.3 Trig Functions

Let $P(t) = (x, y)$. Then,

$\sin(t) = y$, $\cos(t) = x$

"sine" "cosine"



tangent: $\tan t = \frac{\sin t}{\cos t} = \frac{y}{x}$, Cotangent: $\cot t = \frac{\cos t}{\sin t} = \frac{x}{y}$

secant: $\sec t = \frac{1}{\cos t} = \frac{1}{x}$, cosecant: $\csc t = \frac{1}{\sin t} = \frac{1}{y}$

Ex] For $t = \frac{\pi}{6}$, find the values of all 6 trig functions

We know $P(\pi/6) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ So,

x y

$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$, $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$, $\tan\left(\frac{\pi}{6}\right) = \frac{\sin \pi/6}{\cos \pi/6} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}}$

$\cot(\pi/6) = \frac{\cos \pi/6}{\sin \pi/6} = \sqrt{3}$, $\sec \frac{\pi}{6} = \frac{1}{\cos \pi/6} = \frac{1}{\sqrt{3}/2} = 2/\sqrt{3}$

$\csc \pi/6 = \frac{1}{\sin \pi/6} = \frac{1}{1/2} = 2.$

Ex) What happens at compass points?

$t = \pi/2$, $P(\pi/2) = (0,1)$

$\sin \pi/2 = 1$, $\cos \pi/2 = 0$, $\tan \pi/2 = \frac{\sin \pi/2}{\cos \pi/2} = \frac{1}{0}$ = undefined

$\cot \pi/2 = \frac{\cos \pi/2}{\sin \pi/2} = \frac{0}{1} = 0$

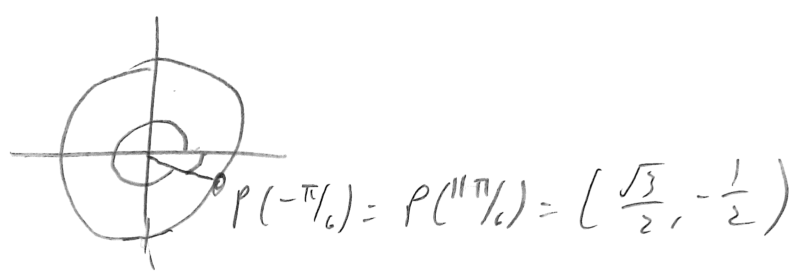
$\sec \pi/2 = \frac{1}{\cos \pi/2} = \frac{1}{0}$ = und. $\qquad \csc \pi/2 = \frac{1}{\sin \pi/2} = \frac{1}{1} = 1$.

see table 6.5

Remark) Coterminal angles give the same values.

Suppose $t_1 = -\pi/6$ rad. $t_2 = 11\pi/6$ rad

t_1 and t_2 are coterminal

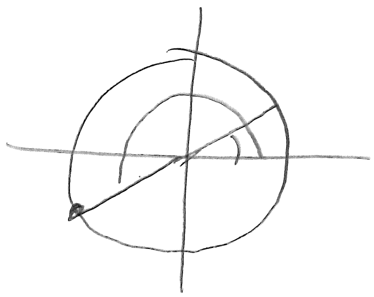


Note) Quadrants are still useful

II: sine + , cosine −	I: sine + , cosine +
III: sine − , cosine −	IV: sine − , cosine +

Reference angles are also still useful!

Ex] For $t = 7\pi/6$, find all 6 trig functions



$P(\pi/6) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\downarrow$

$P(7\pi/6) = \left(\frac{-\sqrt{3}}{2}, -\frac{1}{2}\right)$

$\sin(7\pi/6) = -\frac{1}{2}$, $\cos 7\pi/6 = -\frac{\sqrt{3}}{2}$, $\tan 7\pi/6 = 1/\sqrt{3}$

$\cot 7\pi/6 = \frac{\cos 7\pi/6}{\sin 7\pi/6} = \sqrt{3}$, $\sec 7\pi/6 = \frac{1}{\cos 7\pi/6} = -\frac{2}{\sqrt{3}}$

$\csc 7\pi/6 = \frac{1}{\sin 7\pi/6} = -2$.

If x, y lie on the unit circle, then $x^2 + y^2 = 1$. We know that $x = \cos t$, $y = \sin t$.

So, $(\cos(t))^2 + (\sin(t))^2 = 1$ (or $\cos^2(t) + \sin^2(t) = 1$)

This is the Pythagorean identity.

Similarly, (1) $\tan^2(t) + 1 = \sec^2(t)$

(2) $1 + \cot^2(t) = \csc^2(t)$

Ex/ If $\sin(t) = 2/3$ and $P(t)$ is in the second quadrant, find $\cos(t)$.

$$\sin^2(t) + \cos^2(t) = 1 \longrightarrow \left(\frac{2}{3}\right)^2 + \cos^2(t) = 1$$

$$\rightarrow \frac{4}{9} + \cos^2(t) = 1$$

$$\rightarrow \cos^2(t) = \frac{5}{9}$$

$$\rightarrow \cos(t) = \pm\frac{\sqrt{5}}{3}$$

Since we are in the second quadrant,
$\cos(t) = -\frac{\sqrt{5}}{3}$.