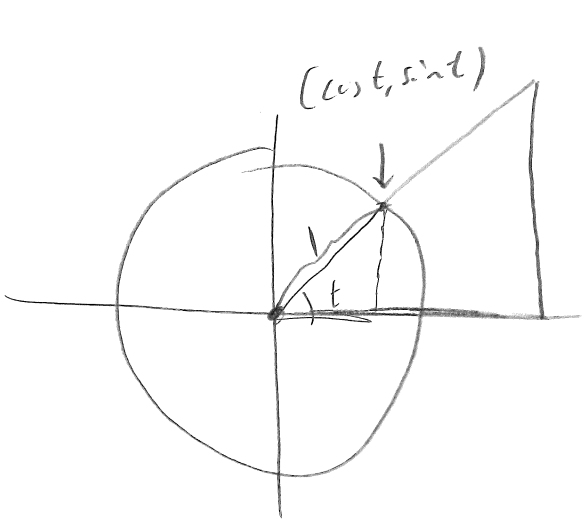
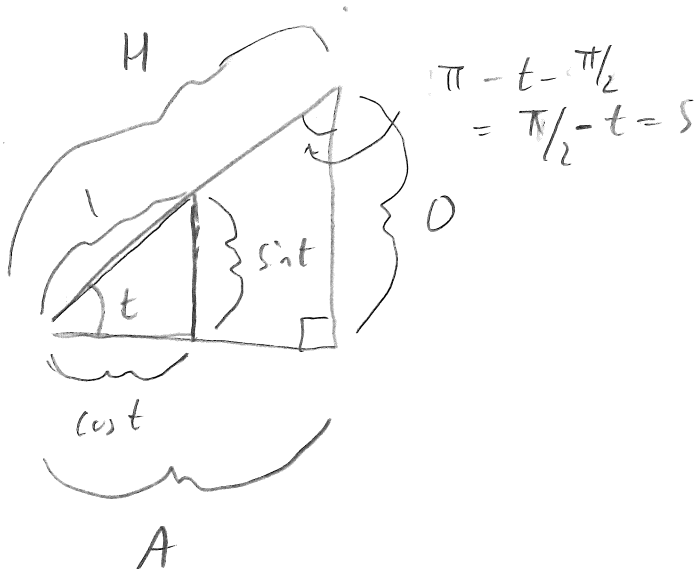
6C.4 Right Triangle Trig

Given a right triangle, how to we find side lengths?





These are similar triangles!
with scaling of H.

In other words, $\frac{A}{\cos t} = \frac{H}{1} \longrightarrow \cos t = \frac{A}{H}$

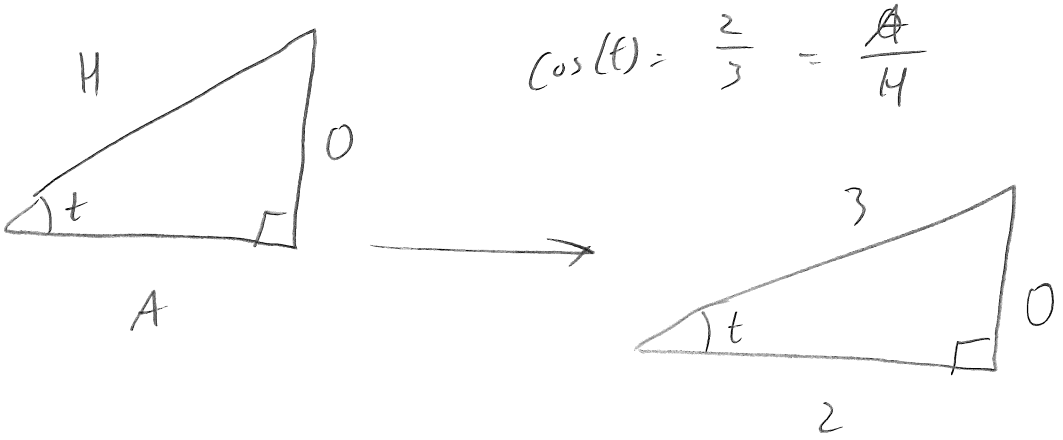
Similarly, $\sin t = \frac{O}{H}$, $\tan t = \frac{O}{A}$

$\cot t = \frac{A}{O}$, $\sec t = \frac{H}{A}$, $\csc t = \frac{H}{O}$

Mnemonic: SOHCAHTOA

Sine is O/H, cosine is A/H, tangent is O/A

Ex $\cos t = \frac{2}{3}$ Find the side lengths of a right triangle with angle t.



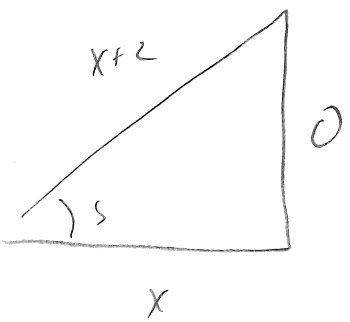
$\cos(t) = \frac{2}{3} = \frac{A}{H}$

$A^2 + O^2 = H^2 \longrightarrow O = \sqrt{3^2 - 2^2} = \sqrt{5}$

Q: Are there other side lengths that work?

A: Yes - any values A and H such that $\frac{A}{H}$ simplifies to $\frac{2}{3}$.

Ex $\cos(s) = \frac{x}{x+2}$ for some $x > 0$, find $\cot(s)$.



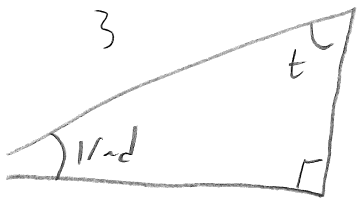
$\cos(s) = \frac{x}{x+2} = \frac{A}{H}$

$A^2 + O^2 = H^2 \longrightarrow x^2 + O^2 = (x+2)^2$

$O^2 = 4x + 4$

$O = \sqrt{4x+4} = 2\sqrt{x+1}$.

Ex]



Find A, B, t.

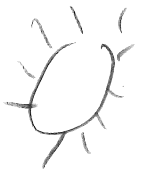
$$t = \pi - \frac{\pi}{2} - 1 = \frac{\pi}{2} - 1 \approx .57 \text{ rad}$$

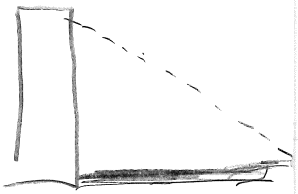
$$\frac{O}{H} = \sin(1) = \frac{B}{3} \longrightarrow B = 3\sin(1) \approx 2.52$$

$$\frac{A}{H} = \cos(1) = \frac{A}{3} \longrightarrow A = 3\cos(1) \approx 1.62$$

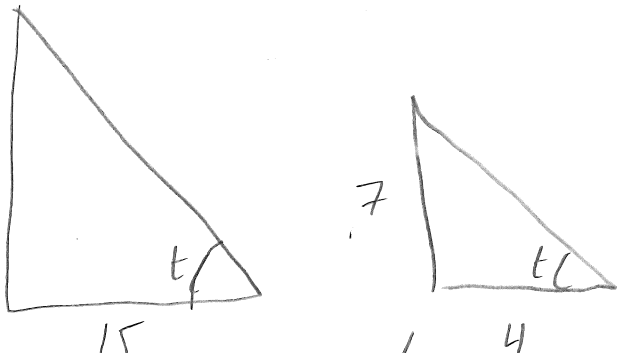
Ex] A 7 ft pole casts a shadow of 4 ft at the same time a tower casts a shadow of 15 ft. How tall is the tower?

Draw a picture!





These are similar triangles



tower

pole

$$\frac{O}{A} = \tan t = \frac{7}{4}$$

$$\frac{O}{A} = \tan t = \frac{H}{15}$$

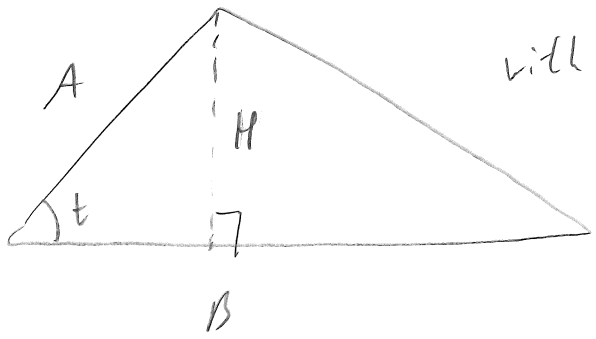
$$\longrightarrow \frac{7}{4} = \frac{H}{15}$$

$$H = 26.25 \text{ ft.}$$

Ex] Area of a triangle

If base and height are known, $A = \frac{1}{2}BH$.

But, what if you have something like



with A, B, t known.

Then, $\sin(t) = \frac{H}{A} \rightarrow A\sin(t) = H$

$\rightarrow A = \frac{1}{2}B(A\sin(t)) = \frac{1}{2}AB\sin(t)$