7.1 Graphs of Sine and Cosine

$\sin(\frac{\pi}{2}) = \sin(\frac{\pi}{2} + 2\pi) = \sin(\frac{\pi}{2} + 2 \cdot 2\pi) = \ldots$

Def A <u>periodic function</u> f is such that

$f(x) = f(x+p)$ for some $p > 0$.

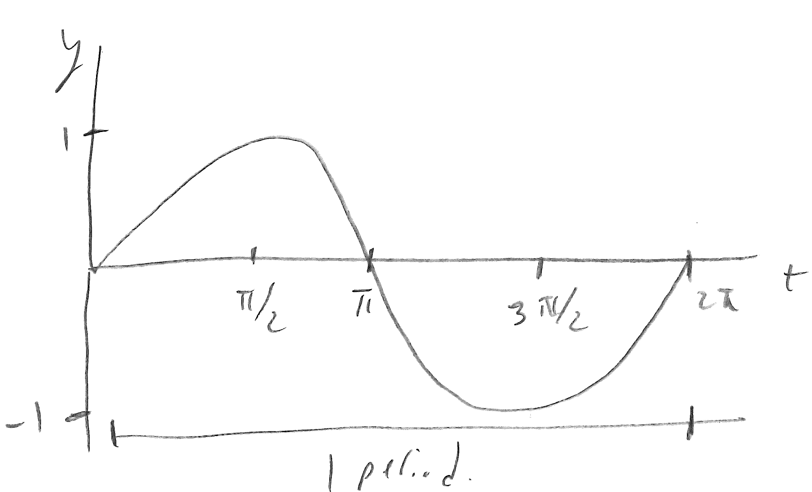
The <u>period</u> of f is the smallest such p such that $f(x) = f(x+p)$.

Sine and cosine are periodic with period 2π.

<u>Graphing the sine function</u> (graphing cosine is similar - see the textbook for more details)

$P(t) = (\cos(t), \sin(t))$

t	0	$\pi/6$	$\pi/4$	$\pi/6$	...	$5\pi/3$	$7\pi/4$	$11\pi/6$	2π
$\sin t$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$		$-\sqrt{3}/2$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	0



See desmos animation.

Extrema of $\sin(t)$.

Max of 1, min of -1. Where do these occur?

$$1 = \sin\left(\tfrac{\pi}{2}\right) = \sin\left(\tfrac{\pi}{2} + 2\pi\right) = \ldots$$
$$-1 = \sin\left(-\tfrac{\pi}{2}\right) = \sin\left(\tfrac{\pi}{2} + 2\pi\right) = \ldots$$

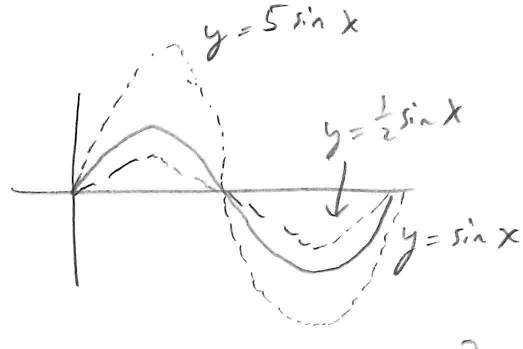
max at $t = \frac{\pi}{2} + 2k\pi$, ~~if~~ for all integers k.
min at $t = -\frac{\pi}{2} + 2k\pi$

Def) The amplitude of $y = A\sin x$ (or $A\cos x$) is $|A|$.
(absolute value)

Why? The max height of $y = \sin x$ is 1.
$A \cdot \sin(x)$ means we need to multiply 1 by $|A|$.

The amplitude A vertically stretches or compresses the sine function

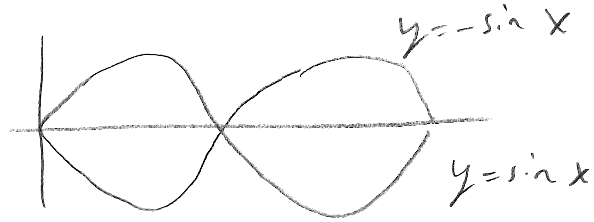
Ex



Q: What happens if $A < 0$?

A: Graph is reflected about x-axis.

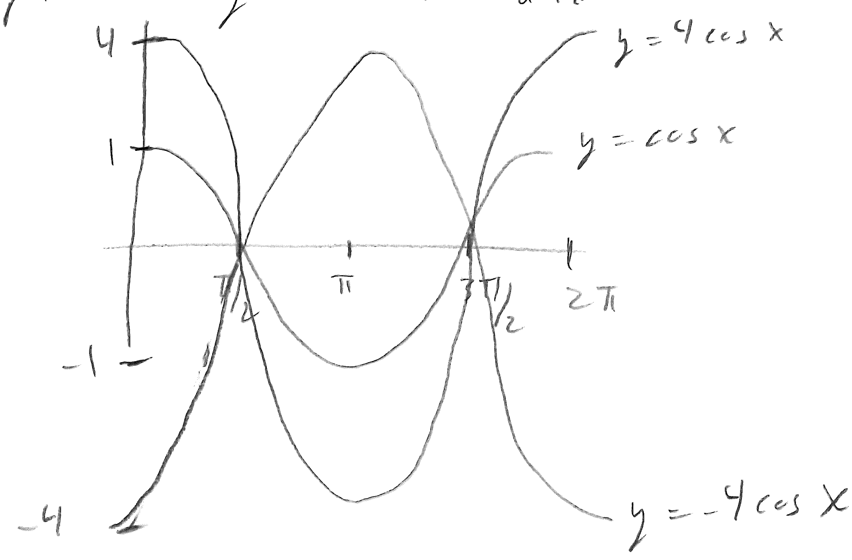
Ex)



Note: Changing the amplitude does not change the period.

Def/Note: A sine wave is the graph of a function of the form $y = A\sin(B(x-C))$ or $y = A\cos(B(x-C))$.

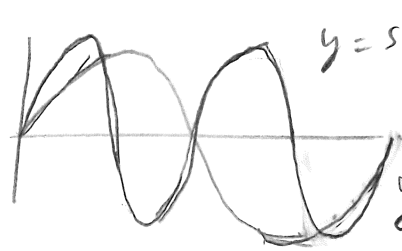
Ex) plot $y = 4\cos x$ and $-4\cos x$



For $\sin(Bx)$ (or $\cos(Bx)$), the B changes the period of $\sin x$ from 2π to $\frac{2\pi}{B}$.

Changing B horizontally stretches or compresses $y = A\sin Bx$ (or $A\cos Bx$) by a factor of B

Ex)

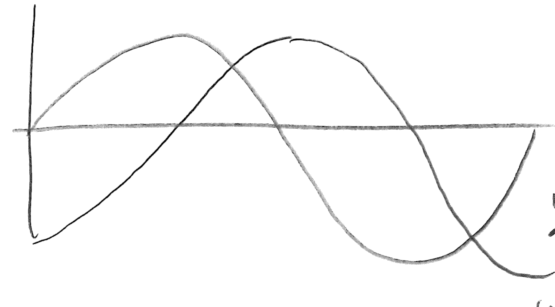


When $B < 0$, the graph is reflected about the y-axis.

Def) The phase shift of $y = A\sin B(x-C)$ is C.
(or $A\cos B(x-C)$)

C controls the horizontal shift of the graph.

Ex)



Note: $C > 0 \rightarrow$ shift left, $C < 0 \rightarrow$ shift right

putting it all together

Ex] Find the amplitude, period, and phase shift of $y = 3\sin\left(2x - \frac{4}{3}\pi\right)$

First, note that $y = 3\sin\left(2\left(x - \frac{2}{3}\pi\right)\right)$

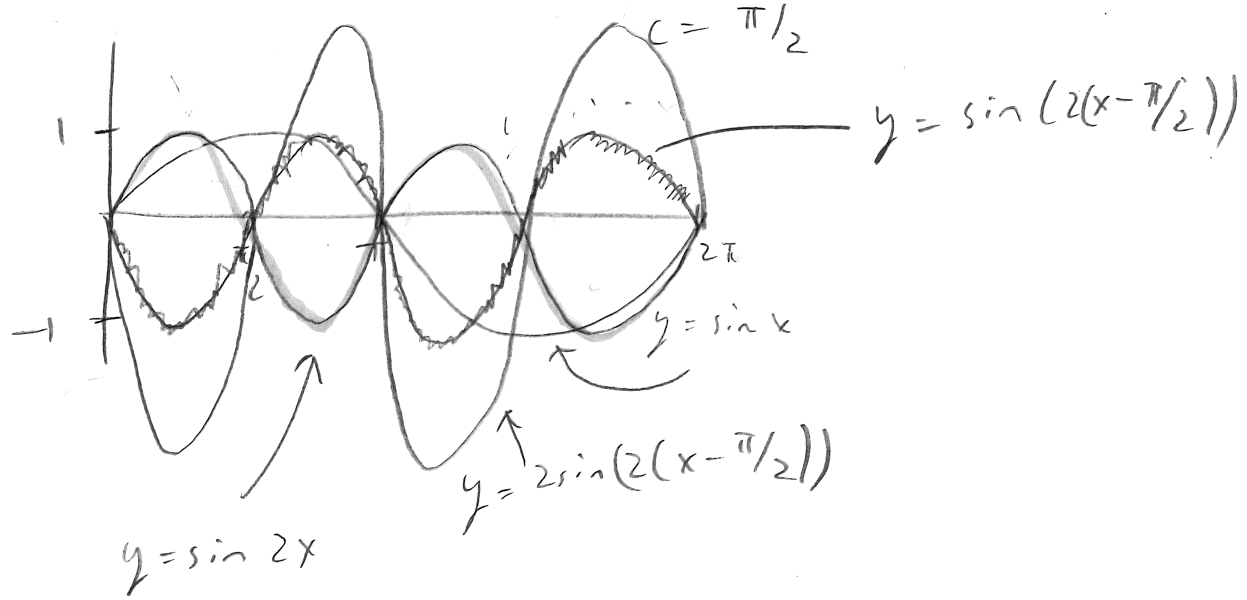
$$A = 3, \quad \text{period} = \frac{2\pi}{B} = \frac{2\pi}{2} = \pi$$

$$C = \frac{2}{3}\pi.$$

Ex] Graph $y = 2\sin\left(2\left(x - \frac{\pi}{2}\right)\right)$

$A = 2$, $B = 2 \rightarrow \text{period} = \frac{2\pi}{2} = \pi$

$C = \pi/2$



Recall

$f(x)$ is even if $f(-x) = f(x)$
odd if $f(-x) = -f(x)$

Notice that cosine is even, and sine is odd.

In other words, $\sin(-x) = -\sin(x)$
$\cos(-x) = \cos(x)$