Section 8.1

TRIG IDENTITIES

Recall: Elementary Identities

$$\sec(x) = \frac{1}{\cos(x)} \quad \text{and} \quad \cos(x) = \frac{1}{\sec(x)}$$

$$\csc(x) = \frac{1}{\sin(x)} \quad \text{and} \quad \sin(x) = \frac{1}{\csc(x)}$$

$$\tan(x) = \frac{1}{\cot(x)} \quad \text{and} \quad \cot(x) = \frac{1}{\tan(x)}$$

$$\tan(x) = \frac{\sin(x)}{\cos(x)} \quad \text{and} \quad \cot(x) = \frac{1}{\tan(x)}$$

What do we mean by identities anyways?

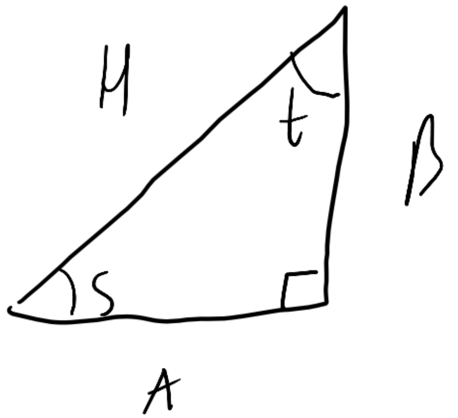
Pythagorean Identity: $\sin^2(x) + \cos^2(x) = 1$

We can show this is true by using the unit circle, and then use it to calculate other things.

Ex: Verify $\tan(t) = \frac{1}{\cot(t)}$

$$\frac{1}{\cot(t)} = \frac{1}{\cos(t)/\sin(t)} = \frac{\sin(t)}{\cos(t)} = \tan t$$

Cofunction Identities



$$\pi = \frac{\pi}{2} + s + t$$

$$s = \frac{\pi}{2} - t$$

$$\sin(s) = \frac{B}{H}, \quad \cos(s) = \frac{A}{H}$$

$$\sin(t) = \frac{A}{H}, \quad \cos(t) = \frac{B}{H}$$

$$\sin\left(\frac{\pi}{2} - t\right) = \cos(t)$$

$$\cos\left(\frac{\pi}{2} - t\right) = \sin(t)$$

Cofunction Identities

$\cos x = \sin\left(\frac{\pi}{2} - x\right)$	$\sin x = \cos\left(\frac{\pi}{2} - x\right)$
$\tan x = \cot\left(\frac{\pi}{2} - x\right)$	$\cot x = \tan\left(\frac{\pi}{2} - x\right)$
$\sec x = \csc\left(\frac{\pi}{2} - x\right)$	$\csc x = \sec\left(\frac{\pi}{2} - x\right)$

Ex: Find all six trig functions for $\frac{\pi}{2} - t$ if $\sin(t) = \frac{3}{5}$, and t is in the first quadrant

$$A^2 + 3^2 = 5^2$$

$$A = 4$$

$$\sin(t) = \frac{3}{5} \longrightarrow \cos\left(\frac{\pi}{2} - t\right) = \frac{3}{5}$$

$$\tan(t) = \frac{3}{4} \longrightarrow \cot\left(\frac{\pi}{2} - t\right) = \frac{3}{4}$$

Recall: Even/Odd functions

Even functions: $f(x) = f(-x)$ (symmetric about y-axis)

Odd functions: $-f(x) = f(-x)$ (sym. about origin)

Even/odd identities

Cosine and Secant are Even, the remaining trig functions are Odd

$\cos(x) = f(x)$

Ex: Is $g(x) = \sec(x)\tan(x)$ even or odd?

$$\begin{aligned} g(-x) &= \sec(-x)\tan(-x) \\ &= \sec(x)(-\tan(x)) \\ &= -\sec(x)\tan(x) \\ &= -g(x) \end{aligned}$$

$\longrightarrow$ g is odd

Verifying Identities Example

Show that $2\sec^2(t) = \underbrace{\frac{1}{1+\sin(t)} + \frac{1}{1-\sin(t)}}_{RHS}$

$$RHS: \frac{1}{1+\sin(t)} + \frac{1}{1-\sin(t)} = \frac{1-\sin(t)}{(1+\sin(t))(1-\sin(t))} + \frac{1+\sin(t)}{(1+\sin(t))(1-\sin(t))}$$

$$= \frac{1-\sin(t)}{1-\sin^2(t)} + \frac{1+\sin(t)}{1-\sin^2(t)} = \frac{2}{1-\sin^2(t)}$$

$$\cos^2 t + \sin^2 t = 1 \Rightarrow \quad = \frac{2}{\cos^2(t)} = 2\sec^2(t)$$

$$\sec(t) = \frac{1}{\cos t}$$

Section 8.2

SUM AND DIFFERENCE FORMULAS

Sum and Difference Formulas for Sine and Cosine

$+$ $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$-$ $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

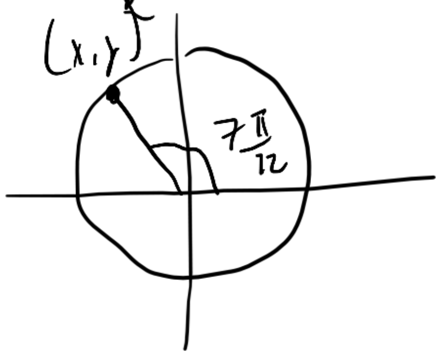
$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

Can be summarized as:

$$\sin(\alpha\pm\beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha\pm\beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

Ex: Calculate $\sin\left(\frac{7\pi}{12}\right)$ in exact form



$$\frac{7\pi}{12} = \frac{3\pi}{12} + \frac{4\pi}{12} = \frac{\pi}{4} + \frac{\pi}{3}$$

$$\sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right) = \sin\frac{\pi}{4}\cos\frac{\pi}{3} + \cos\frac{\pi}{4}\sin\frac{\pi}{3}$$

$$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{\sqrt{2}+\sqrt{6}}{4}$$

$$\frac{\pi}{3} \sim \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$$

$$\frac{\pi}{4} \sim \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

Sum and Difference Formulas for Tangent

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

$$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

$$\cot(\alpha+\beta)=\frac{\cot\alpha\cot\beta-1}{\cot\alpha+\cot\beta}$$

$$\cot(\alpha-\beta)=\frac{\cot\alpha\cot\beta+1}{\cot\alpha-\cot\beta}$$

Ex: Calculate $\tan\left(\frac{7\pi}{12}\right)$ in exact form

$\alpha = \frac{\pi}{3}, \quad \beta = \frac{\pi}{4}$

$$= \frac{1+\sqrt{3}}{1-\sqrt{3}}$$

Ex: Verify $\tan(\alpha+\beta) = \frac{1+\tan(\beta)\cot(\alpha)}{\cot(\alpha)-\tan(\beta)}$

$$\tan \alpha + \beta = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta} \cdot \frac{(1/\tan\alpha)}{(1/\tan\alpha)} = \frac{\frac{\tan\alpha}{\tan\alpha} + \frac{\tan\beta}{\tan\alpha}}{\frac{1}{\tan\alpha} - \frac{\tan\alpha\tan\beta}{\tan\alpha}}$$

$$= \frac{1+\cot\alpha\tan\beta}{\cot\alpha - \tan\beta}$$

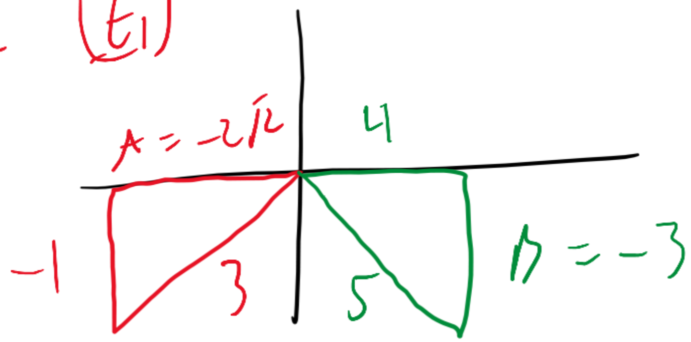
What about angles not in the 1st quadrant?

Ex: Let $\sin(t_1) = -\frac{1}{3}$ and $\cos(t_2) = 4/5$, with $\underbrace{\pi < t_1 < \frac{3\pi}{2}}_{Q3}$ and $\underbrace{\frac{3\pi}{2} < t_2 < 2\pi}_{Q4}$. Find $\sin(t_1 + t_2)$

$A^2 + (-1)^2 = 3^2$

$A = \pm 2\sqrt{2}$

(t_1)



$$= \sin t_1 \cos t_2 + \cos t_1 \sin t_2$$

$$= \left(-\frac{1}{3}\right)\left(\frac{4}{5}\right) + \left(-\frac{2\sqrt{2}}{3}\right)\left(-\frac{3}{5}\right)$$

$$= \frac{-4 + 6\sqrt{2}}{15}$$

As an exercise, find $\cos(t_1 + t_2)$

Product to Sum formulas

$$\sin\alpha\cos\beta = \frac{1}{2}\sin(\alpha+\beta) + \frac{1}{2}\sin(\alpha-\beta)$$

$$\cos\alpha\cos\beta = \frac{1}{2}\cos(\alpha+\beta) + \frac{1}{2}\cos(\alpha-\beta)$$

$$\sin\alpha\sin\beta = \frac{1}{2}\cos(\alpha+\beta) - \frac{1}{2}\cos(\alpha-\beta)$$

Ex: Find $\sin(52.5°)\cos(7.5°)$

$$= \frac{1}{2}\left(\sin(52.5+7.5) + \sin(52.5-7.5)\right)$$

$$= \frac{1}{2}\left(\sin(60°) + \sin(45°)\right)$$

$$= \frac{1}{2}\left(\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\right) = \frac{\sqrt{3}+\sqrt{2}}{4}$$

Sum to product formulas

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

Ex: Find $\sin(75°) + \sin(15°)$

$$= 2\sin\left(\frac{75+15}{2}\right)\cos\left(\frac{75-15}{2}\right)$$

$$= 2\sin(45)\cos(30)$$

$$= 2\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{6}}{2}$$

Section 8.3

DOUBLE AND HALF ANGLE FORMULAS

Double Angle formulas

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$\begin{aligned}\cos(2\theta) &= 2\cos^2\theta - 1\\ &= 1 - 2\sin^2\theta\\ &= \cos^2\theta - \sin^2\theta\end{aligned}$$

Ex: $\sin(t) = \frac{5}{6}, \tan(t) < 0$

$\tan t = \frac{\sin t}{\cos t}$

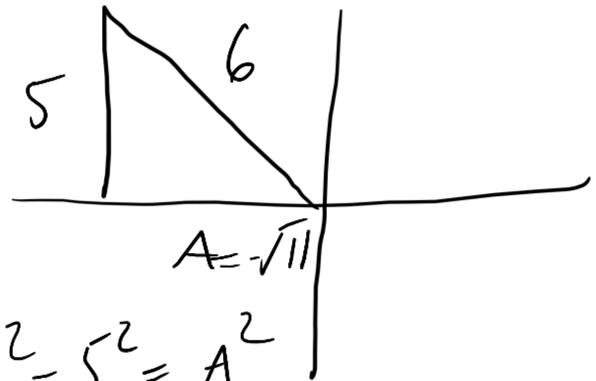
$\hookrightarrow, \cos t < 0$

(1) Sketch a diagram

(2) Find $\sin(2t)$ and $\cos(2t)$

(3) What quadrant is angle $2t$ in?

(1)



$6^2 - 5^2 = A^2$

$A = -\sqrt{11}$

(2) $\sin(2t) = 2 \sin t \cos t$

$= 2\left(\frac{5}{6}\right)\left(-\frac{\sqrt{11}}{6}\right)$

$= \frac{-5\sqrt{11}}{18}$

$\cos(2t) = \cos^2 t - \sin^2 t = \ldots = -\frac{14}{36} = \frac{-7}{18}$

$\cos t = \frac{-\sqrt{11}}{6} \qquad \sin t = \frac{5}{6}$

(3): $\sin 2t, \cos 2t < 0 \rightarrow$ III quadrant

Half Angle formulas

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

More examples in class on Wednesday!