Section 8.4

INVERSE TRIG FUNCTIONS

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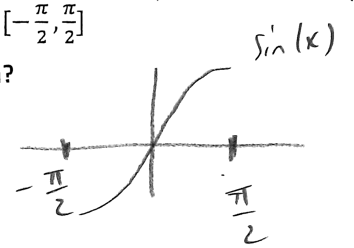
Arcsine

Definition: The arcsine function, written as arcsin(x) or $\sin^{-1} x$ is the inverse of sine, restricted to the domain of $[-\frac{\pi}{2}, \frac{\pi}{2}]$

Why this domain?



needs to pass horizontal line test

In other words,

$\arcsin(x) =$ θ if and only if $\sin(\theta) =$ x AND $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

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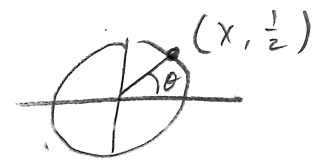
Example: Finding inverse sine values

Determine the exact value of the following

(1) $\sin^{-1}(\frac{1}{2}) =$ θ

$\sin\theta = \frac{1}{2}$ and $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

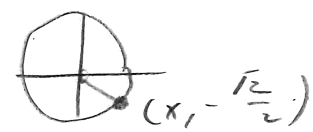
$\theta = \frac{\pi}{6}$



(2) $\sin^{-1}(-\frac{\sqrt{2}}{2}) =$ θ

$\sin\theta = -\frac{\sqrt{2}}{2}$ and $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$\theta = -\frac{\pi}{4}$



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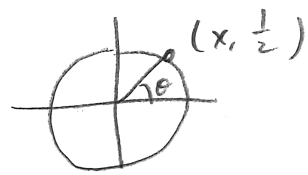
Example: Function composition

Determine the exact value of the following

(1) $\sin\left(\arcsin\left(\frac{1}{2}\right)\right) =$ $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

$\arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$



(2) $\sin^{-1}(\sin\left(\frac{\pi}{4}\right)) =$ $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

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Example continued

(3) $\cancel{\sin(\sin^{-1}}(\frac{4\pi}{3})) =$
$\sin^{-1}(\sin($

$\sin\frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$

$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \theta \rightarrow \sin\theta = -\frac{\sqrt{3}}{2}$

and $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$

Notice that

$\sin(\arcsin(x)) = x$ for all x $\longrightarrow \theta = -\frac{\pi}{3}$

$\arcsin(\sin(x)) = x$ for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

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Arccosine

Definition: The arccosine function, written as arccos(x) or $\cos^{-1} x$ is the inverse of sine, restricted to the domain of $[0, \pi]$

Why this domain?

needs to pass horizontal line test

In other words,

$\arccos(x) =$ θ if and only if $\cos(\theta) =$ x AND $0 \leq \theta \leq \pi$

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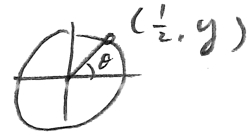
Example: Finding inverse sine values

Determine the exact value of the following

(1) $\cos^{-1}(\frac{1}{2}) = \theta$

$\cos\theta = \frac{1}{2}$ and $0 \le \theta \le \pi$

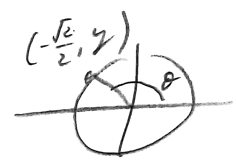
$\theta = \frac{\pi}{3}$



(2) $\arccos(-\frac{\sqrt{2}}{2}) = \theta$

$\cos\theta = \frac{-\sqrt{2}}{2}$ and $0 \le \theta \le \pi$

$\theta = \frac{3\pi}{4}$



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Example: Function composition

Determine the exact value of the following

(1) $\cos\left(\arccos\left(\frac{1}{2}\right)\right) = \cos\frac{\pi}{3} = \frac{1}{2}$

$\arccos(\frac{1}{2}) = \theta \rightarrow \cos\theta = \frac{1}{2}, 0 \le \theta \le \pi \rightarrow \theta = \frac{\pi}{3}$

(2) $\cos^{-1}(\cos\left(\frac{\pi}{3}\right)) = \cos^{-1}(\frac{1}{2}) = \frac{\pi}{3}$

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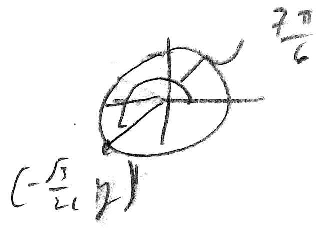
Example continued

(3) $\cos^{-1}(\cos(\frac{7\pi}{6})) =$ $\cos^{-1}(-\frac{\sqrt{3}}{2})$

$= \frac{5\pi}{6}$



Notice that

$\cos(\arccos(x)) = x$ for all x

$\arccos(\cos(x)) = x$ for $0 \le x \le \pi$

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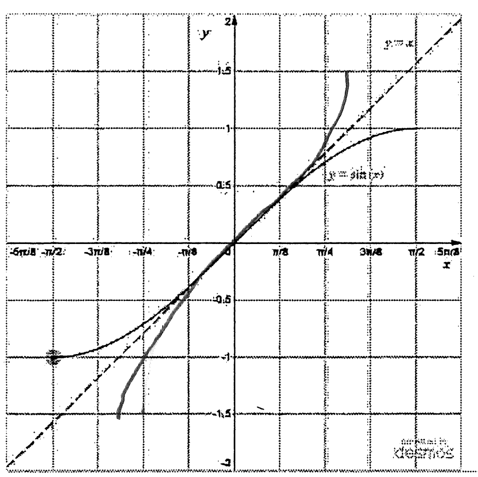
Graphing arcsine



What does $y_2 = \arcsin(x)$ look like?

Domain: $[-1, 1]$

Range: $[-\frac{\pi}{2}, \frac{\pi}{2}]$

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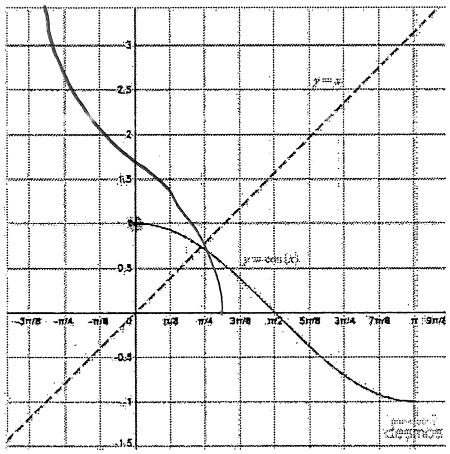
Graphing arccosine



What does $y_2 = \arccos(x)$ look like?

Domain: $[-1, 1]$

Range: $[0, \pi]$

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Arctangent

Definition: The arctangent function, written as arctan(x) or $\tan^{-1} x$ is the inverse of tangent, restricted to the domain of $(-\frac{\pi}{2}, \frac{\pi}{2})$

Why this domain?

horizontal line test

In other words,

$\arctan(x) =$ θ if and only if $\tan(\theta) =$ x AND $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

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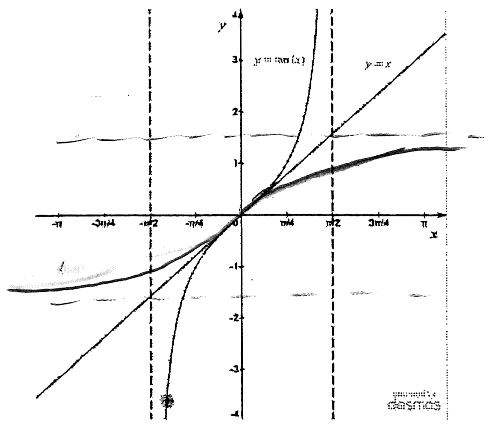
Graphing arctangent



What does $y_2 = \arctan(x)$ look like?

Domain: $(-\infty, \infty)$

Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

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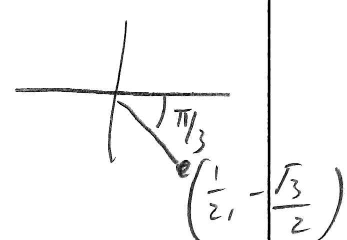
Example: Other inverses

Determine the exact value of the following

(1) $\arctan(-\sqrt{3}) = \theta$

$\tan\theta = -\sqrt{3}$ and $-\frac{\pi}{2} < \theta \ \frac{\pi}{2}$

$\theta = -\frac{\pi}{3}$



(2) $\operatorname{arccsc}(-2) = \theta$

$\csc\theta = -2$ and $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

↳ $\sin\theta = -\frac{1}{2} \longrightarrow \theta = -\frac{\pi}{6}$

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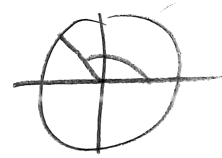
Example: More function compositions

Determine the exact value of the following

(1) $\sin\left(\arccos\left(-\frac{1}{2}\right)\right) =$ $\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$



$\cos\theta = -\frac{1}{2} \rightarrow \theta = \frac{2\pi}{3}$

(2) $\operatorname{arccot}\left(\tan\left(-\frac{\pi}{3}\right)\right) =$ $\operatorname{arccot}(-\sqrt{3}) = \theta$

$\hookrightarrow \cot\theta = -\sqrt{3} \rightarrow \tan\theta = -\frac{1}{\sqrt{3}}$

$\theta = 5\pi/6.$

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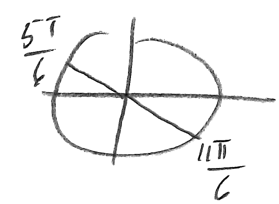
Example: Trig Equations

Find the solutions of the below equations on the interval $[0,2\pi]$

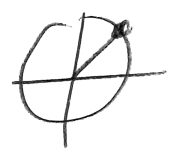
(1) $\sin(x) = -\frac{1}{\sqrt{3}}\cos(x)$ $\rightarrow \tan\theta = -\frac{1}{\sqrt{3}}$

$\theta = \frac{5\pi}{6}, \frac{11\pi}{6}$



(2) $\arctan(x) = \frac{\pi}{4}$

$\tan\frac{\pi}{4} = x \rightarrow x = 1$



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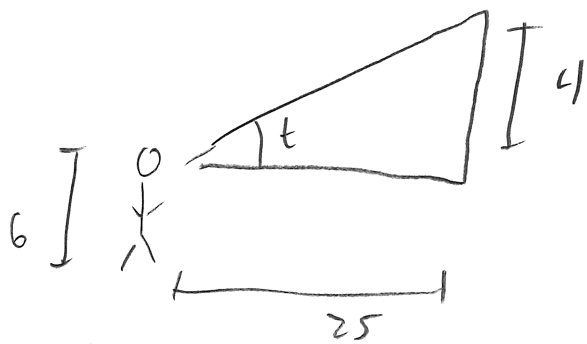
Example: Application

The rim of a standard basketball goal is 10 feet high. A point guards eyes are 6 feet above the floor. She stands 25 feet from a point directly below the front of the rim. At what angle t (measured in degrees) must she incline her eyes to look directly at the front of the rim?



$$\tan(t) = \frac{4}{25}$$

$$t = \tan^{-1}\left(\frac{4}{25}\right)$$

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