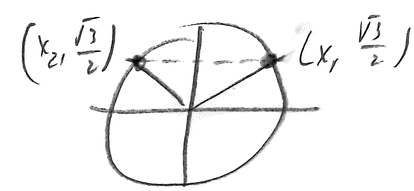
Section 8.5

SOLVING TRIGONOMETRIC EQUATIONS

Example 1

Consider the equation $2\sin(t) = \sqrt{3}$.

$\sin(t) = \frac{\sqrt{3}}{2}$



(1) Find the solutions to the equation on $[0,2\pi]$.

$t = \frac{\pi}{3}, \frac{2\pi}{3}$ (use reference angles to see that if $t_1 = \frac{\pi}{3}$, $t_2 = \pi - t_1 = \frac{2\pi}{3}$)

(2) Find all solutions

$\sin(t) = \sin(t + 2\pi k)$

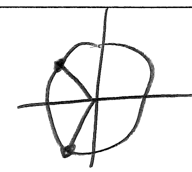
So, $t = \frac{\pi}{3} + 2\pi k, \frac{2\pi}{3} + 2\pi k$

Example 2

Consider the equation $4\cos(t) = -2$.

$\cos(t) = -\frac{1}{2}$

(1) Find the solutions to the equation on $[0,2\pi]$.



$t = \frac{2\pi}{3}, \frac{4\pi}{3}$

(2) Find the solutions to the equation on $[-\pi,\pi]$.

$t = \frac{2\pi}{3}, -\frac{2\pi}{3}$ b/c $\frac{2\pi}{3}$ is fine, but need to shift $\frac{4\pi}{3}$ by 2π

Example 2 continued

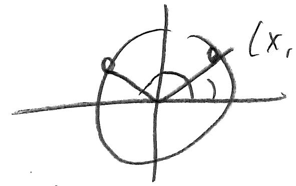
Consider the equation $4\cos(t) = -2$.

(3) Find all solutions

$t = \frac{2\pi}{3} + 2\pi k, \frac{4\pi}{3} + 2\pi k$

Example 3

Find all solutions of the equation $\sin(t) = .312$.



$t_1 = \arcsin(.312) + 2\pi k$

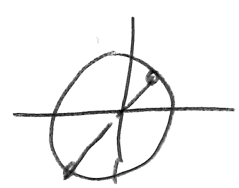
$t_2 = \pi - t_1 = \pi - \arcsin(.312) + 2\pi k$

Example 4

Find all solutions of the equation $\tan(t) = \sqrt{3}$.

$\tan(t) = \frac{y}{x} = \sqrt{3} \rightarrow x = \frac{1}{2}, y = \frac{\sqrt{3}}{2}$
or $x = -\frac{1}{2}, y = -\frac{\sqrt{3}}{2}$

$[0, 2\pi]: \; t = \frac{\pi}{3}, \frac{4\pi}{3}$

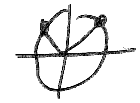


$(-\infty, \infty): \; t = \frac{\pi}{3} + 2\pi k, \; \frac{4\pi}{3} + 2\pi k$

Notice this is the same as $t = \frac{\pi}{3} + \pi k$.

Example 5

(1) Find all solutions of the equation $\sin(2t) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$



Suppose we just had $\sin(s) = \frac{\sqrt{2}}{2}$. Then, $s = \frac{\pi}{4} + 2\pi k$

$\frac{3\pi}{4} + 2\pi k$.

Notice that $\sin\left(2 \cdot \frac{\pi}{8}\right) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

This motivates the following. For $\sin(2t) = \frac{\sqrt{2}}{2}$,

$t = \frac{s}{2} = \frac{\pi}{8} + \pi k, \; \frac{3\pi}{8} + \pi k$

we expect π (not 2π) here because the period of

[redacted]

$\sin(2t)$ is π.

Example 5 continued

(2) Write down all solutions that are within the interval $[0,2\pi]$.

$\frac{\pi}{8}$ is contained in the interval. With period $= \pi$,

we need to check $\frac{\pi}{8} + \pi$. Is this in the interval? Yes.

How about $\frac{\pi}{8} + 2\pi$? No.

So, $t = \frac{\pi}{8}, \frac{9\pi}{8}$. Similarly, we also have

$\frac{3\pi}{8}$ and $\frac{3\pi}{8} + \pi = \frac{11\pi}{8}$.

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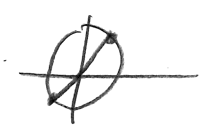
Example 6

(1) Find all solutions of the equation $\cot(3t) = \sqrt{3}$. $\rightarrow$ $\tan(3t) = \frac{1}{\sqrt{3}}$



Consider instead $\tan(s) = \frac{1}{\sqrt{3}} \longrightarrow s = \frac{\pi}{6}, \frac{7\pi}{6}$

Dividing by 3 gives $t = \frac{s}{3} = \frac{\frac{\pi}{6} + 2\pi k}{3}, \frac{\frac{7\pi}{6} + 2\pi k}{3}$

$= \frac{\pi}{18} + \frac{2\pi k}{3}, \frac{7\pi}{18} + \frac{2\pi k}{3}$

Example 6 continued

(2) Write down all solutions that are within the interval $[0,2\pi]$.

Check possible k: $k=0$: $t = \frac{\pi}{18}, \frac{7\pi}{18}$ both in interval

$k=1$: $t = \frac{13\pi}{18}, \frac{19\pi}{18}$ both in interval

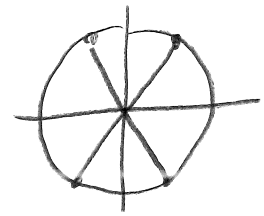
$k=2$: $t = \frac{25\pi}{18}, \frac{31\pi}{18}$ both in interval

$k=3$: not in interval.

So, $t = \frac{\pi}{18}, \frac{7\pi}{18}, \frac{13\pi}{18}, \frac{19\pi}{18}, \frac{25\pi}{18}, \frac{31\pi}{18}$.

Example 7

Solve the equation $4\sin^2(x) = 3$. $\sin(x) = \pm\frac{\sqrt{3}}{2}$



on $[0, 2\pi] \longrightarrow x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

on $(-\infty, \infty)$: $x = \frac{\pi}{3} + 2\pi k$, $\frac{4\pi}{3} + 2\pi k$

$\frac{2\pi}{3} + 2\pi k$, $\frac{5\pi}{3} + 2\pi k$

Notice that this simplifies to

$x = \frac{\pi}{3} + \pi k$, $\frac{2\pi}{3} + \pi k$

Example 8

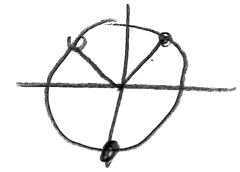
Solve the equation $\sin(2x) = .3\cos(x)$.

$\sin(2x) = 2\sin(x)\cos(x)$. So,

$0 = 2\sin(x)\cos(x) - .3\cos(x) = \cos(x)(2\sin(x) - .3)$

Either $\cos(x) = 0$ or $2\sin(x) - .3 = 0$

$\sin(x) = .15$



$x = \frac{\pi}{2} + \pi k$

$x = \sin^{-1}(.15) + 2\pi k$

$\pi - \sin^{-1}(.15) + 2\pi k$

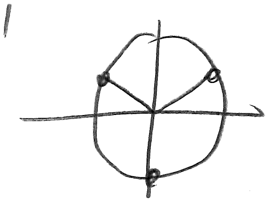
Example 9

Solve the equation $2\sin^2(x) + \sin(x) = 1$. $\longrightarrow$ $2\sin^2(x) + \sin(x) - 1 = 0$

$\rightarrow (2\sin(x) - 1)(\sin(x) + 1) = 0$

Either $2\sin(x) - 1 = 0$ or $\sin(x) + 1 = 0$

$\sin(x) = \frac{1}{2}$ $\qquad$ $\sin(x) = -1$



On $[0, 2\pi]$: $x = \frac{\pi}{6}, \frac{5\pi}{6}$ $\qquad$ $x = \frac{3\pi}{2}$

On $(-\infty, \infty)$: $x = \frac{\pi}{6} + 2\pi k, \frac{5\pi}{6} + 2\pi k, \frac{3\pi}{2} + 2\pi k$

Example 10

Solve the equation $\cos(2x)\left[\sqrt{3}\tan(2x) - 1\right] = 0$.

Either $\cos(2x) = 0$ or $\sqrt{3}\tan(2x) - 1 = 0$

$\tan(2x) = \frac{1}{\sqrt{3}}$

Consider instead $\cos(s) = 0$

$s = \frac{\pi}{2} + 2\pi k, \frac{3\pi}{2} + 2\pi k$

$x = \frac{s}{2} = \boxed{\frac{\pi}{4} + \pi k, \frac{3\pi}{4} + \pi k}$

Consider instead $\tan(s) = \frac{1}{\sqrt{3}}$

From Ex 6, $s = \frac{\pi}{6} + 2\pi k, \frac{7\pi}{6} + 2\pi k$

$x = \frac{s}{2} = \boxed{\frac{\pi}{12} + \pi k, \frac{7\pi}{12} + \pi k.}$