Section 3.1: Linear Functions

The Linear Function

A linear function has the general form:

$$y = mx + b$$

Where:

- m is the **slope**, representing the function's constant **rate of change**.
- b is the **y-intercept**, representing the **initial value** of the function.

Key Notes:

- The average rate of change of a linear function is always constant.
- The graph of a linear function is a straight line.

Forms of a Line

Recall the formula for slope: $m = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}}$.

There are two common forms for the equation of a line:

- **Slope-Intercept Form:** $y = mx + b$
- **Point-Slope Form:** $y - y_1 = m(x - x_1)$

Secant Lines

The slope of a secant line between two points on a curve represents the **average rate of change** between those points.

Example. Let $f(x) = x^2$. Find the equation of the secant line between $a = 2$ and $b = 4$.

1. **Find the slope (average rate of change):**

$$\begin{aligned} m = \frac{f(b) - f(a)}{b - a} &= \frac{f(4) - f(2)}{4 - 2} \\ &= \frac{4^2 - 2^2}{4 - 2} \\ &= \frac{16 - 4}{2} = \frac{12}{2} = 6 \end{aligned}$$

2. **Use the point-slope form:** First, find a point on the line. We can use the point $(a, f(a))$:

$$(2, f(2)) = (2, 2^2) = (2, 4)$$

Now, plug the slope and the point into the point-slope formula:

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 4 &= 6(x - 2) \end{aligned}$$

3. **(Optional) Convert to slope-intercept form:**

$$y - 4 = 6x - 12$$
$$y = 6x - 8$$

Proportionality

Definition 1. A quantity A is **proportional** to a quantity B if $A = kB$ for some constant k. The constant k is called the **constant of proportionality**.