Section 3.2: Exponential Functions

Introduction to Exponential Growth

Example. Suppose there are 1000 cells in a petri dish, and the number of cells doubles every hour. Let $N(t)$ be the number of cells after t hours.

Time (t)	Number of Cells N(t)
0	1000
1	$1000 \times 2 = 2000$
2	$1000 \times 2 \times 2 = 4000$
3	$1000 \times 2 \times 2 \times 2 = 8000$

This pattern leads to the formula:

$$N(t) = 1000 \times 2^t$$

Definition 1. An **exponential function** is a function of the form

$$f(x) = ab^x$$

where a is the **initial value** ($a \neq 0$) and b is the **base** ($b > 0, b \neq 1$).

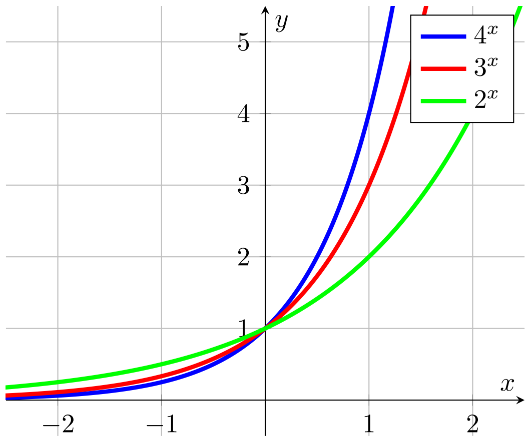
A key property of exponential functions is that for every unit increase in x, the function value is multiplied by the base: $f(x+1) = b \cdot f(x)$.

Graphs of Exponential Functions

Exponential Growth ($b > 1$)

When the base b is greater than 1, the function grows. As the base increases, the graph becomes steeper for $x > 0$.

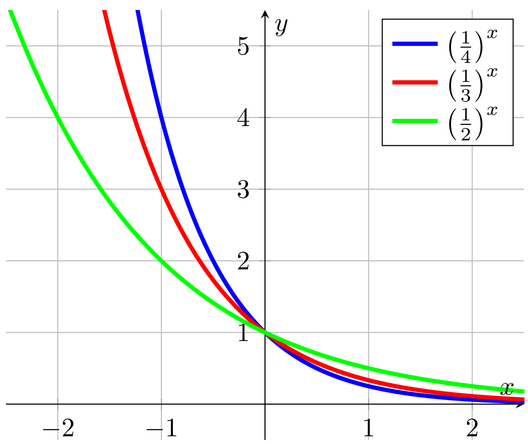
- The function is **increasing** and **concave up**.
- As $x \to \infty$, $f(x) \to \infty$.
- As $x \to -\infty$, $f(x) \to 0$.



Exponential Decay $(0 < b < 1)$

When the base b is between 0 and 1, the function decays, or gets smaller.

- The function is **decreasing** and **concave up**.
- As $x \to \infty$, $f(x) \to 0$.
- As $x \to -\infty$, $f(x) \to \infty$.



The Natural Exponential Function

Definition 2. The **natural exponential function** is the exponential function with the base e, where $e \approx 2.71828....$ This function is written as:

$$f(x) = e^x$$