Section 8.3: Double and Half Angle Formulas

Definition 1 (Double Angle Formulas)**.**

$$\begin{aligned}\sin(2\theta) &= 2\sin\theta\cos\theta \\ \cos(2\theta) &= \cos^2\theta - \sin^2\theta \\ &= 2\cos^2\theta - 1 \\ &= 1 - 2\sin^2\theta \\ \tan(2\theta) &= \frac{2\tan\theta}{1-\tan^2\theta}\end{aligned}$$

Example. Given $\sin(t) = \frac{5}{6}$ and $\tan(t) < 0$.

1. Draw a picture.
2. **Find** $\cos(t)$ **and Quadrant:** Since $\sin(t)$ is positive and $\tan(t)$ is negative, t must be in **Quadrant II**. In QII, $\cos(t)$ is negative. $A^2 + 5^2 = 6^2 \implies A^2 = 36 - 25 = 11 \implies A = \pm\sqrt{11}$. So, $\cos(t) = -\frac{\sqrt{11}}{6}$.
3. **Find** $\sin(2t)$ **and** $\cos(2t)$**:**

$$\begin{aligned}\sin(2t) &= 2\sin(t)\cos(t) \\ &= 2\left(\frac{5}{6}\right)\left(-\frac{\sqrt{11}}{6}\right) \\ &= \frac{-10\sqrt{11}}{36} = -\frac{5\sqrt{11}}{18}\end{aligned}$$

$$\begin{aligned}\cos(2t) &= \cos^2(t) - \sin^2(t) \\ &= \left(-\frac{\sqrt{11}}{6}\right)^2 - \left(\frac{5}{6}\right)^2 \\ &= \frac{11}{36} - \frac{25}{36} = -\frac{14}{36} = -\frac{7}{18}\end{aligned}$$

4. **What quadrant is** $2t$ **in?** Since $\sin(2t)$ is negative and $\cos(2t)$ is negative, the angle $2t$ is in **Quadrant III**.

Definition 2 (Half Angle Formulas)**.**

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$$
$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$$
$$\tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

(To determine the sign, look to see which quadrant the angle $\frac{\theta}{2}$ is in)

Example. Find the exact value of $\sin(15°)$ using a half-angle identity. Notice that since $15°$ is in the first quadrant, we will take the positive root.

$$\begin{aligned}\sin(15°) &= \sqrt{\frac{1-\cos(30°)}{2}} \\ &= \sqrt{\frac{1-\sqrt{3}/2}{2}} \\ &= \frac{\sqrt{2-\sqrt{3}}}{2}\end{aligned}$$